Skein relations. Conway polynomial

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Surgery "transfer"

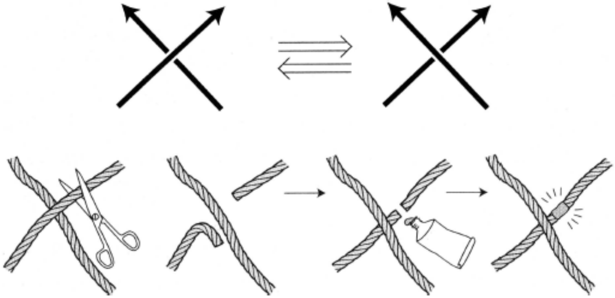


Figure : Careful: it can change the type of a knot!

Surgery "resolution"

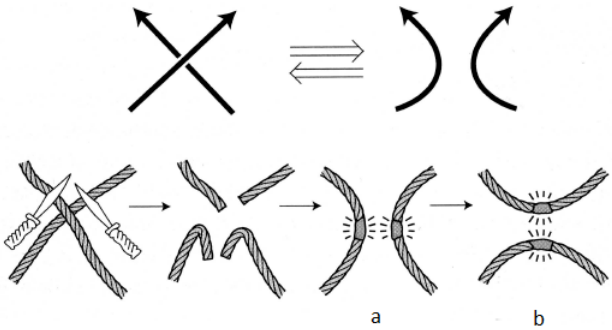


Figure : Careful: it also can change the type of a knot!

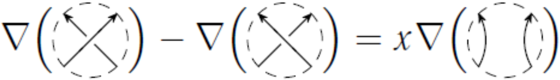
Defining relations

Definition

A Conway polynomial of a link is a function ∇ giving for any diagram D a polynomial $\nabla(D)$ in one variable x defined by the following two relations, called skein relations:

If links N and N' are equivalent, we require $\nabla(N) = \nabla(N')$.

$$\nabla(\bigcirc) = 1$$



Examples

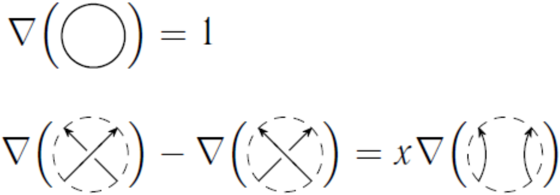
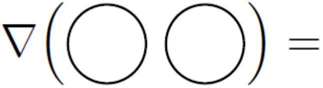


Figure : Conway relations



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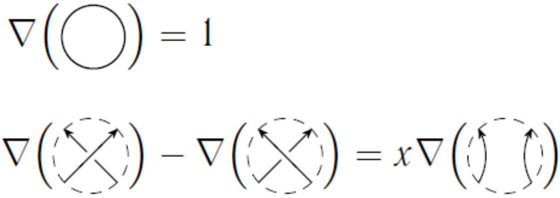
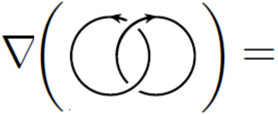


Figure : Conway relations



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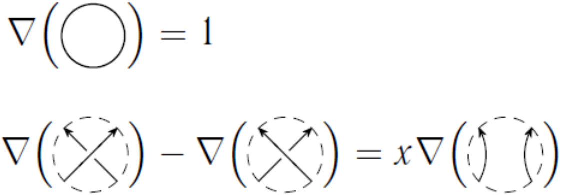
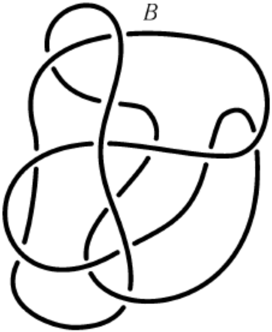


Figure : Conway relations

Conway invariant is nice

$$\nabla(A) = 1 - x^2$$



$$\nabla(B) = x^2 + 1$$

Can we really compute Conway polynomial?

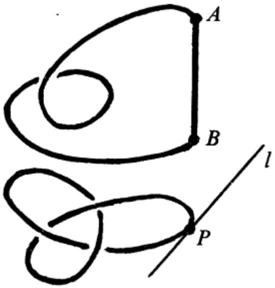
Theorem

Yes, we can really compute Conway polynomial!

Lemma

Any knot can be surgically changed to be an unknot.

Proof:



Lemma

For any split link L we have $\nabla(L) = 0$.

Corollary

We can compute Conway polynomial!

HOMFLY polynomial

Most popular name: HOMFLY

H = Hoste, O = Ocneanu, M = Millet, F = Freyd, L = Lickorish, Y = Yetter

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L = Lickorish, Y = Yetter, M = Millet, P = Przytycki, H = Hoste, O = Ocneanu, T = Traczik, U = unknown discoverers

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Figure : Skein relation for HOMFLY polynomial

Is HOMFLY the biggest guy in the neighborhood?

No, HOMFLY can't distinguish between the following knots: